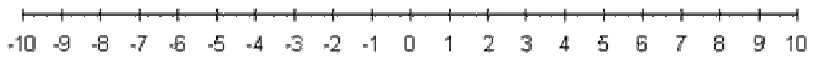
6 Integers

1 The negative numbers.

There are many situations in which you need to use numbers below zero, one of these is temperature, others are money that you can deposit (positive) or withdraw (negative) in a bank, steps that you can take forwards (positive) or backwards (negative).

Positive integers are all the whole numbers greater than zero: 1, 2, 3, 4, 5, ... Negative integers are all the opposites of these whole numbers: -1, -2, -3, -4, -5, ….

The Number Line
The number line is a line labelled with the integers in increasing order from left to right, that extends in both directions:



For any two different places on the number line, the integer on the right is greater than the integer on the left.

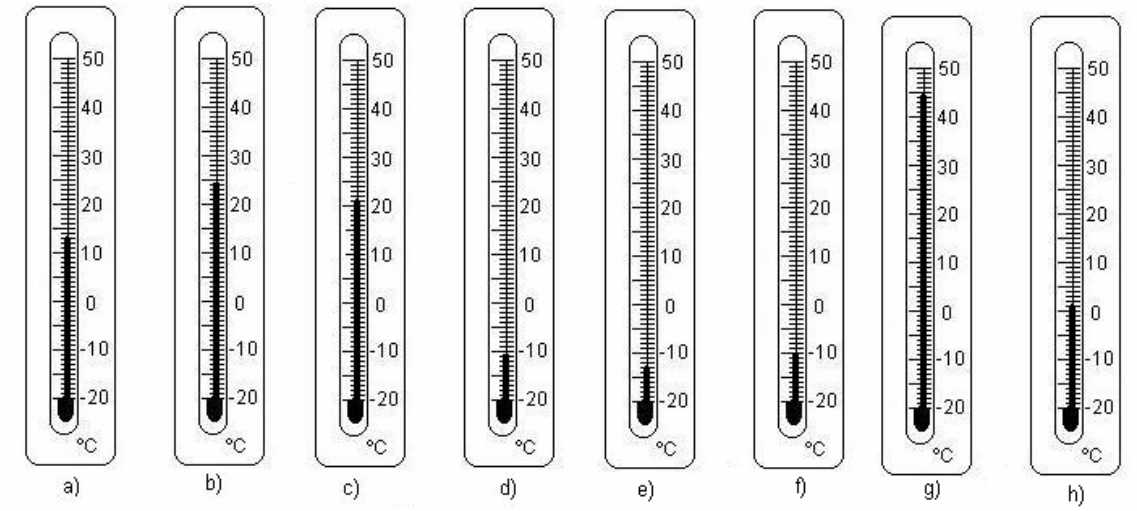
Examples:

9 > 4 Is read: nine is **'greater than'** four -7 < 9 Is read: minus seven is **'less than'** nine.

Exercise 1

a) Read: 6 > -9 -2 > -8 0 > -5

b) Write down the temperatures shown on these thermometers. Find their position in a number line.



Exercise 2 What is the temperature which is:

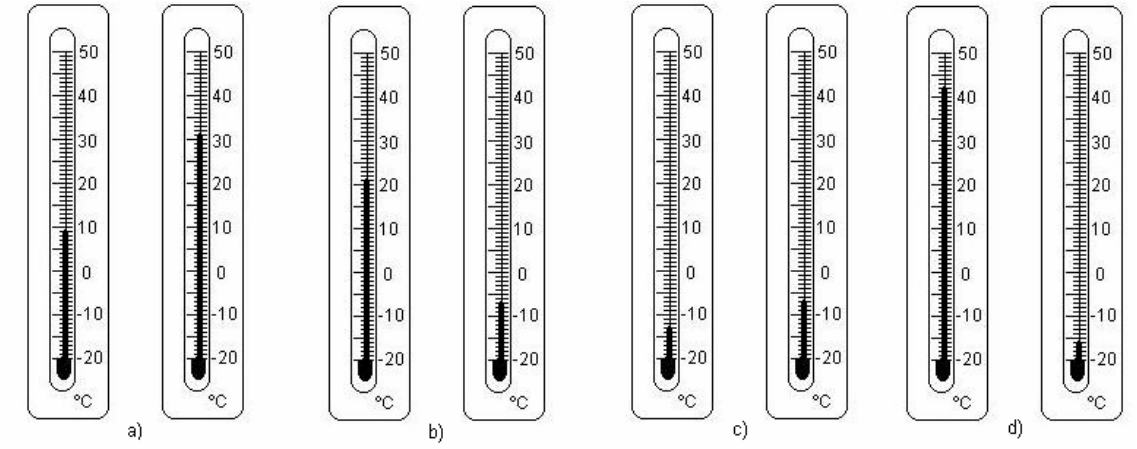
a) 7 degrees lower than 5º C

b) 6 degrees lower than 4º C

c) 16 degrees higher than - 4º C

d) 9 degrees lower than - 6º C

Exercise 3 What is the difference in temperature between each pair of thermometers?



Exercise 4 Write using integers.

a) Four degrees above zero

b) A withdrawal of 20.000 €

c) 250 meters below sea level

d) Three degrees below zero

e) A deposit of $200.00

f) An elevation of 8848 meters above sea level

g) A gain of 19kg

Exercise 5 Write the opposite of each integer given above.

2 Absolute Value of an Integer

The absolute value of any number is the distance between that number and zero on the number line.

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If the number is positive, the absolute value is the same number.
If the number is negative, the absolute value is the opposite.

The absolute value of a number is always a positive number (or zero). We specify the absolute value of a number *n* by writing *n* in between two vertical bars: |*n*|.

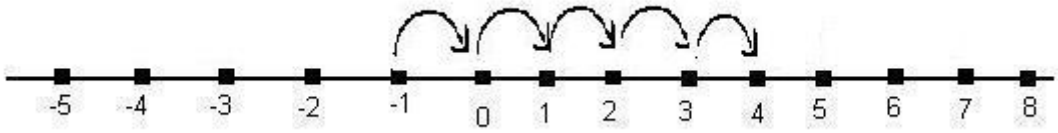
Examples:

|6| = 6 |-10| = 10 |0| = 0 |123| = 123 |-3404| = 3404

3 Adding Integers

There is a way to understand how to add integers. In order to add positive and negative integers, we will imagine that we are moving along a number line.

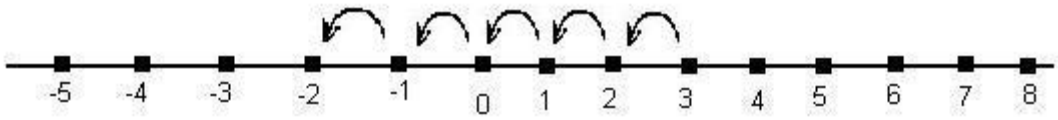
If we want to add -1 and 5, we start by finding the number -1 on the number line, exactly one unit to the left of zero. Then we would move five units to the right. Since we landed four units to the right of zero, the answer must be 4.



If asked to add 3 and -5, we can start by finding the number three on the number line (to the right of zero).

Then we move five units left from there because negative numbers make us move to the left side of the number line.

Since our last position is two units to the left of zero, the answer is -2.



Addition rules

When adding integers with the same sign

We add their absolute values, and give the result the same sign.

Examples:

2 + 5 = 7

(-7) + (-2) = -(7 + 2) = -9

(-80) + (-34) = -(80 + 34) = -114

With the opposite signs

We take their absolute values, subtract the smallest from the largest, and give the result the sign of the integer with the **larger** absolute value.

Example: 8 + (-3) = ?

The absolute values of 8 and -3 are 8 and 3.

Subtracting the smaller from the larger gives 8 - 3 = 5, and since the larger absolute value was 8, we give the result the same sign as 8, so 8 + (-3) = 5.

Example: 8 + (-17) = ?

The absolute values of 8 and -17 are 8 and 17.

Subtracting the smaller from the larger gives 17 - 8 = 9, and since the larger absolute value was 17, we give the result the same sign as –17

So 8 + (-17) = -9.

Example: -22 + 11 = ? $\left.\begin{array}{l}|-22| = 22 \\ |11| = 11\end{array}\right.$

Subtracting the smaller from the larger gives 22 - 11 = 11, and since the larger absolute value was 22, we give the result the same sign as -22

So -22 + 11 = -11.

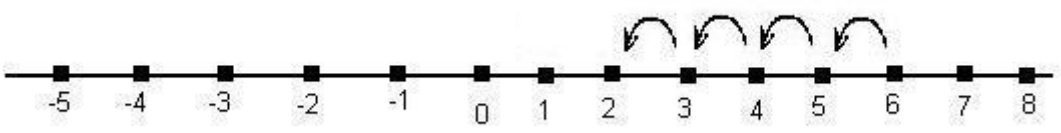
Example: 32 + (-32) = ? $\left.\begin{array}{l}|-32| = 32 \\ |32| = 32\end{array}\right\}$

Subtracting the smaller from the larger, it gives 32 - 32 =0. The sign in this case does not matter, since 0 and -0 are the same. Note that 32 and -32 are opposite integers.

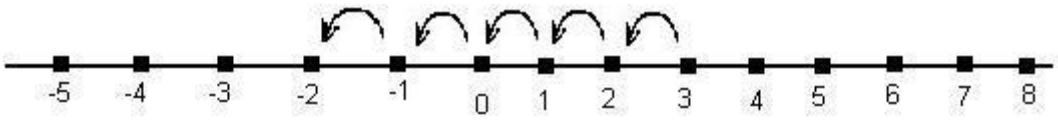
4 Subtracting Integers

To understand how to subtract integers, we will imagine that we are moving along a number line.

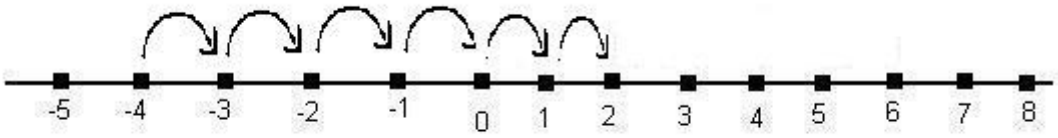
If we have to subtract 6 and 4, we start by finding the number six on the number line (6 units to the right of zero). Then we move four units to the left. Since we land two units to the right of zero, the answer is 2.



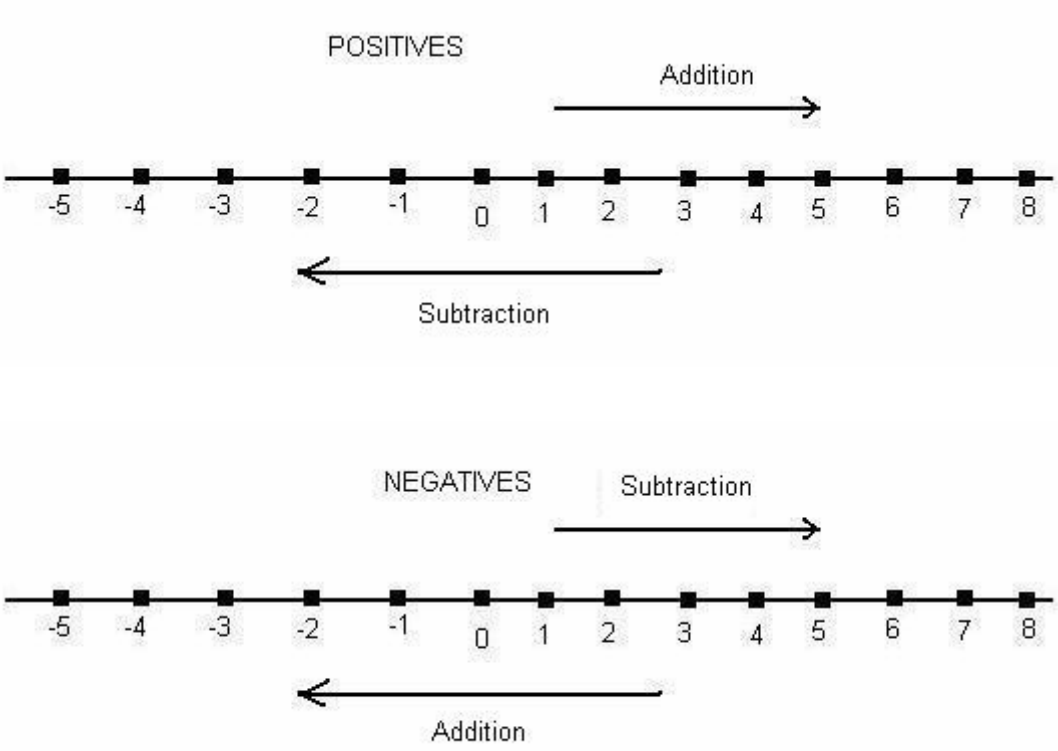
If we have to subtract 3 and 5, first find the number three on the number line, then move five units further left. Since we land two units left of zero, the answer is -2.



If we have to subtract -4 and -6, we start at -4, four units to the left of zero, and then we move six units to the right (**because we subtract a negative number!**). Since we land two units to the right of zero, the answer is 2.



So, to add and subtract positives or negatives numbers using the number line, you must move as is indicated below.



Subtraction rules

Subtracting an integer is the same as adding the opposite.

We convert the subtracted integer to its opposite, and add the two integers.

Examples:

7 - 4 = 7 + (-4) = 3 12 - (-5) = 12 + (5) = 17 -8 - 7 = -8 + (-7) = -15

-22 - (-40) = -22 + (40) = 18

The result of subtracting two integers could be positive or negative.

Exercises

Exercise 6 Write and solve an addition for each of the following sentences.

a) The temperature rises 6 degrees, and then falls 3 degrees.

b) You deposit $50.5 into the bank and then withdraw $38.25

c) The temperature falls 10 degrees, and then falls 4 degrees.

d) You fall 45.3 metres down a mountain, and then climb up 23.5 metres, but you fall again 15 metres.

Exercise 7 Add the following.

a) 5+(-3)=

b) (-12)+(-8)=

c) (-6)+12=

d) 15+(-19)=

e) (-5)+(-6)=

f) 22+(-43)=

g) (-5)+9=

h) 3+5=

i) (-4)+(0)=

Exercise 8

8.1 Add the following.

a) 18+(-20)+6=

b) (-13)+(-9)+(+5)=

c) (-7)+(-5)+(-4)=

d) (-17)+8+(-4)=

e) (8)+(-30)+22=

f) (-6)+5+(-6)=

g) (-9)+7+(-4)+6=

h) 7+(-13)+(-4)+(-5)=

8.2. Remove brackets that are not needed, rewrite the subtraction sentence and solve.

a) (+6)-(+8) =

b) (-4)-(-3) =

c) (+6)-(+9) =

d) (+3)-(-2) =

e) (+4)+(-3)+(-6)-(+7)+(+6) =

f) (-8)-(+5)+(+7)+(-8) =

8.3.Subtract.

a) 6-8 =

b) -7-3 =

c) 4-(-3)=

d) 8-(-8)=

e) -3-6 =

f) -12-8 =

g) -7-(-9)=

h) -6 - 4 =

8.4. Subtract.

a) 5-(-3)= e) -9-(-5)=

b) 5-4= f) 9-(-4)=

c) -8-4= g) -5-3=

d) -3-6= h) -2-(-4)=

8.5. Subtract.

a) –3 + (-4) –2 = b) – 6 - (-8) – 9 =

c) –12 - (-4) + (-3) = d) –7 + (-3) + (-2) =

5 Multiplying Integers

To multiply a pair of integers:

If both numbers have the same sign (positive or negative)

Their product is the product of their absolute values (their product is positive)

These examples help us to understand why a positive number is always the result of multiplying two numbers of the same sign.

Negative times negative: Make a pattern. Look at these sequences and complete them :

Example 1
-2 x 3 = -6
-2 x 2 = -4
-2 x 1 = -2
-2 x 0 = 0
-2 x -1 = 2
-2 x (-2) =
-2x(-3) =
-2x(-4) =

Example 2
-3 x 3 = -9
-3 x 2 = -6
-3 x 1 = -3
-3 x 0 = 0
-3 x (-1) =
-3 x (-2) =
-3 x(-3) =
-3 x(-4) =

Example 3
(-4) x 3 =
(-4) x 2 =
(-4) x 1 =
(-4) x 0 =
(-4) x (-1) =
(-4) x (-2) =
(-4) x (-3) =
(-4) x (-4) =

If the numbers have opposite signs

their product is the *opposite* of the product of their absolute values (their product is negative). If one or both of the integers is 0, the product is 0.

Look at the following chart below.

PRODUCT	+	-
+	POSITIVE	NEGATIVE
-	NEGATIVE	POSITIVE

Note: Multiplying integers of the **same sign** will give a **positive number**.

Multiplying integers with **opposite sign** will give a **negative number.**

Examples:
4× 3 = 12 Both numbers are positive, so we just take their product.

(-4) × (-5) = |-4| × |-5| = 4 × 5 = 20 Both numbers are negative, so we take the product of their absolute values.

In the product of (-7) × 6, the first number is negative and the second is positive, so we take the product of their absolute values, which is |-7| × |6| = 7 × 6 = 42, and give this result a negative sign: -42

So (-7) × 6 = -42

In the product of 12 × (-2), the first number is positive and the second is negative, so we take the product of their absolute values, which is
|12| × |-2| = 12 × 2 = 24, and give this result a negative sign: -24

So 12 × (-2) = -24.

To multiply any number of integers:

1. Count the number of negative numbers in the product.

2. Take the product of their absolute values.

If the number of negative integers counted in step 1 is **even**, the product is just the product from step 2 (**positive).**

If the number of negative integers is **odd,** the product is the opposite of the product in step 2 (give the product in step 2 a **negative** sign).

If any of the integers in the product is 0, the product is 0.

Example: $4 \times (-2) \times 3 \times (-11) \times (-5) = ?$

Counting the number of negative integers in the product, we see that there are 3 negative integers: -2, -11, and -5.
Next, we take the product of the absolute values of each number:
$4 \times |-2| \times 3 \times |-11| \times |-5| = 1320$.
Since there were an odd number of integers, the product is the opposite of 1320, which is –1320

So $4 \times (-2) \times 3 \times (-11) \times (-5) = -1320$.

Example: $-5 \times (-2) \times (-3) \times (-1) \times (4) = ?$

Counting the number of negative integers in the product, we see that there are 4 negative integers

Next, we take the product of the absolute values of each number:
$5 \times 2 \times 3 \times 1 \times 4 = 120$.
Since there is an even number of integers, the product is 120.

Exercise 9 Multiply the following.

a) 5x(-13) =

b) (-12)x(-8) =

c) (-16)x12 =

d) 15x(-19) =

e) (-5)x(-6) =

f) 12x(-3) =

g) (-15)x6 =

h) 3x35 =

i) (-4)x(0)=

k) -3x(-5) =

Exercise 10

10.1 Multiply .

a) 7x(-12)x6=

b) (-13)x(-9)x(+5)=

c) (-3)x(-7)x(-14)=

d) (-6)x8x(-4)=

e) (9)x(-30)x2=

f) (-13)x5x(-6)=

g) (-9)x17x(-4)x26=

h) 4x(-81)x(-4)x(-2)=

10.2. Multiply

a) (+6) ·(+18) =

b) (-12) ·(-3) =

c) (+6) ·(+7) =

d) (+7) ·(-6) =

e) (+3) ·(-3) ·(-2) ·(+6) ·(+3) =

f) (-6) ·(+9) ·(+3) ·(-1) =

10.3. Operate

a) –3 ·[(-4) –2] =

b) – 6 ·[- (-8) – 9] =_

c) [–12 - (-4)] ·(-3) =

d) (–7 + (-3)) ·(-2) =

6 Dividing Integers

To divide a pair of integers the rules are the same as the rules for the product:

If both numbers have the same sign (positive or negative)

Divide the absolute values of the first integer by the absolute value of the second integer (the result is positive)

If the numbers have opposite signs

Divide the absolute value of the first integer by the absolute value of the second integer, and give this result a negative sign.

The chart is.

DIVISION	+	-
+	POSITIVE	NEGATIVE
-	NEGATIVE	POSITIVE

Note: Dividing integers of the **same sign** will give a **positive number**.

Dividing integers with **opposite sign** will give a **negative number.**

Examples:

14 ÷ 2 = 7 Both numbers are positive, so we just divide as usual.

(-24) ÷ (-3) = |-24| ÷ |-3| = 24 ÷ 3 = 8. Both numbers are negative, so we divide the absolute value of the first by the absolute value of the second.

(-100) ÷ 25 The numbers have different signs, so we divide the absolute value of the first number by the absolute value of the second.

|-100| ÷ |25| = 100 ÷ 25 = 4, and give this result a negative sign: -4

So (-100) ÷ 25 = -4

98 ÷ (-7) Both numbers have different signs, so we divide the absolute value of the first number by the absolute value of the second.

|98| ÷ |-7| = 98 ÷ 7 = 14, and give this result a negative sign: -14.

So 98 ÷ (-7) = -14.

Exercise 11 Divide the following.

a) 15:(-3)=

b) (-12):(-4)=

c) (-16):2=

d) 150:(-6)=

e) (-225):(-15)=

f) 12:(-3)=

g) (-156):6=

h) 35:(-35)=

i) (-4):(0)=

Exercise 12

12.1 Divide .

a) 12:(6:(-2))=

b) (-30):[(-15):(+5)]=

c) (-8):[(-14):(-7)]=

d) [(-6)x8]:(-4)=

e) [(9)x(-30)]:2=

12.2. Divide

a) (+76):(+18) =

b) (-12):(-3) =

c) (+42):(+7) =

d) (+72):(-6) =

12.3. Operate

a) –30: [(-4) –2] =

b) – 34 :[(-8) – 9] =

c) [–12 - (-3)]:(-3) =

d) (–7 + (-3)):(-2) =

Order of operations

- Do of all the operations in brackets first.

- Then do multiplication and division in the order they appear, then do addition and subtraction in the order they occur

Easy way to remember them

- Brackets
- Exponents
- Division
- Multiplication
- Addition
- Subtraction
- This gives you **BEDMAS.**

Do one operation at a time.

Example
[10+(-16)+(-24)]/3 = [-6+(-24)]/3 = -30/3= -10

13. Operate

a) (+3):(-3)+(-12):(+6)+ (+3) =

b) 62/[-10+(-4)-4] =

c) $(-3)\cdot(-4)-(-24):6+15:(-3)=$

d) $10:(-2)-(-7)\cdot(-3)+4=$

e) $1+[(-3)/((-5)+2)]-35/(-7)=$

f) $4-[7+(-3)\cdot(-2)]+(-72):(-9)=$

EXTRA EXERCISES

1 Mount Everest is 29,028 feet above sea level. The Dead Sea is 1,312 feet below sea level. What is the difference of altitude between these two points?

2 The temperature in Chicago was 4º C at two in the afternoon. If the temperature dropped 12º C at midnight, what is the temperature now?

3 A submarine was situated 2100 feet below sea level. If it ascends 1230 feet, what is its new position?

4 Aristotle was born in 384 B.C. and died in 322 BC. How old was he when he died?

5 A submarine was situated 1230 feet below sea level. If it descends 125 feet, what is its new position?

6 This is the three-day forecast for Yellowknife (Canada) from the 24th of November 2008

Today	Tue	Wed
Nov 24	Nov 25	Nov 26
		
Snow	Cloudy	Snow
-6°	**-7°**	**-6°**
-13°	**-8°**	**-14°**

What is the difference between the maximum and the minimum temperatures each day?

What are the maximum and the minimum temperatures during these three days?

7 This is the three-day forecast for Birmingham, UK from the 24th of November 2008

Today	Tue	Wed
Nov 24	Nov 25	Nov 26
		
Rain	Sunny	Partly Cloudy
13° 2°	14° -2°	14° 1°

What is the difference between the maximum and the minimum temperatures each day?

8 The Punic Wars began in 264 B.C. and ended in 146 B.C. How long did the Punic Wars last?

9 This is a table with the melting points of some metals

Metal	Melting point º C	Boiling point º C
Aluminium	660.32	2519
Iron	1538	2861
Gold	1064.18	2856
Mercury	-38.83	656.73

a) **Calculate the difference between the melting and the boiling point of each metal**

b) **How much warmer is the melting point of mercury than the melting point of iron?**

10 On the 2nd of January, the temperature dropped from 3ºC at two o'clock in the afternoon to -11ºC at 8 a. m. the next day. How many degrees did the temperature fall?

11 A Greek treasure was buried in the year 164 BC and found in 1843 AD. How long was the treasure hidden?

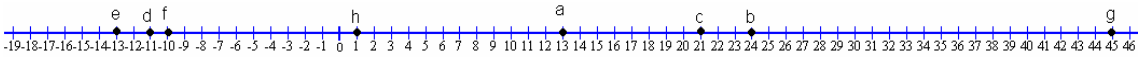
12 On the 1st of December, the level of the water in a reservoir was 130 cm above its average level. On the 1st of July it was 110 cm below its average level. How many cm did the water level drop in this time?

Solutions

Exercise 1

a) Six is greater than minus nine, minus two is greater than minus eight, zero is greater than minus five.

b) a) 13º C, b) 24º C, c) 21º C, d) -11º C, e) -13 º C, f) -10º C, g) 45º C, h) 1º C.



Exercise 2 a) -2ºC; b) -2ºC; c) 14º C; d) - 15º C

Exercise 3 (Right thermometer temperature minus left thermometer temperature) a) 22º; b) -28º; c) 6º; d) -58º.

Exercise 4 a) 4; b) -20.000; c) -250; d) -3; e) 200; f) 8848; g) 19.

Exercise 5 a) -4; b) 20.000; c) 250; d) 3; e) -200; f) -8848; g) -19.

Exercises

Exercise 6 a) $+6-3=3$, b) $50.5-38.25=12.25$, c) $-10-4=-14$, d) $-45.3+23.5-15=-36.8$. **Exercise 7** a) 2, b) -20, c) +6, d) -4, e) -11, f) -21, g) 4, h) 8, i) -4.

Exercise 8

8.1 a) 4, b) -17, c) -16, d) -13, e) 0, f) -7, g) 0, h) -15,

8.2. a) $+6-8=-2$, b) $-4+3=-1$, c) $6-9=-3$, d) $3+2=5$, e) $4-3-6-7+6=-6$, f) $-8-5+7-8=-14$

8.3. a) -2, b) -10, c) 7, d) 16, e) -9, f) -20, g) 2, h) -10.

8.4. a) 8, b) 1, c) -12, d) -9, e) -4, f) 13, g) -8, h) 2.

8.5. a) – 9, b) – 7, c) –11, d) – 12.

Exercise 9 a) -65, b) 96, c) -192, d) -285, e) 30, f) -36, g) -90, h) 105, i) 0, k) 15.

Exercise 10

10.1 a) – 504, b) 585, c) – 294, d) 192, e) – 540, f) 390, g) 15912, h) – 2592,

10.2. a) 108, b) 36, c) 42, d) – 42, e) 324, f) 162.

10.3. a) 18, b) 6, c) 24, d) 20.

Exercise 11 a) – 5, b) 3, c) – 8, d) – 25, e) 15, f) – 4, g) – 26, h) – 1, i) It is a forbidden calculation.

Exercise 12

12.1 a) – 4, b) 10, c) – 4, d) 12, e) – 135,

12.2. a) 4.2, It is not a integer, b) 4, c) 6, d) – 12.

12.3. a) 5, b) 2, c) 3, d) 5.

13. a) 0, b) – 3.4, It is not a integer, c) 11, d) – 22, e) 7, f) – 1.

EXTRA EXERCISES

1 30340; **2** – 8 ºC; **3** 870 feet below sea level; **4** 62 years old; **5** 1355 feet below sea level; **6** Nov 24: 7º, nov 25: 1º, nov 26: 8º ; maximum -6º, minimum -14º **7** Nov 24: 11º, nov 25: 16º, nov 26: 13º; **8** 118 years; **9** a) Boiling point minus melting point: Aluminium 1858.68º, iron 1323º, gold 1791.82º mercury 695.56; b) – 1576.83 (It is negative because the melting point of mercury is cooler than the melting point of iron); **10** 14 ºC; **11** 2007 years; **12** 240 cm.